PRE AP-CALCULUS Name:_____

THE MYSTERY FUNCTION

The following is a table of values for a mystery function, **h**.

x	h(x)
1	0.0000000
2	0.3010300
3	0.4771213
4	0.6020600
5	0.6989700
6	0.7781513
7	0.8450980
8	0.9030900
9	0.9542425
10	1.0000000
11	1.0413927
12	1.0791812
16	1.2041200
20	1.3010300
21	1.3222193
22	1.3424227
30	1.4771213
31	1.4913617
32	1.5051500
40	1.6020600
41	1.6127839
42	1.6232493
50	1.6989700
60	?
70	1.8450980
80	1.9030900
90	?
100	2.0000000
200	2.3010300
1000	3.0000000
10000	?

NOTE: The values in this chart have been rounded to the 7th decimal place.

Your job is to figure what this mystery function is all about. What are its properties? Is it related to any other function(s) you know? Answering the following questions should help you figure out what's going on.

1. By examining the table, try to figure out or at least approximate the three missing values. If you can't get them, don't worry about it and move on to the next question (but come back to this question).

2. Answer the following questions using the table of values and your general mathematical knowledge.

a. What is h(4)? What is h(40)? How are h(4) and h(40) related? If you don't see any relationship now, go onto the next problem and come back.

b. What is h(5)? What is h(50)? How are h(5) and h(50) related?

c. Does this relation also work for other values of x, such as 3 and 30, 2 and 20, etc?

d. Look at h(2) and h(200). Based on this trend, can you figure out what h(400) is going to be?

e. i. Think of 20 as 2 x 10. How are h(20) and h(10) and h(2) related?

ii. Now think of 20 as 4 x 5. How are h(20) and h(4) and h(5) related?

f. Write 24 as a product in 3 different ways. Using each of these ways, predict the value of h(24). Are the values the same for each? Show work.

g. Look back at a. – f. Make a generalization about the relationship between h(ab), h(a), and h(b).

3. We're going to now explore a slightly different relationship.

a. Note that 9 can be written as 3 x 3 or 3^2. Do you see any relationship between h(9) and h(3)?	b. Note that 16 can be written as 4 x 4 or 4^2. How are h(16) and h(4) related?
c. Is this pattern consistent with the value for h(100) found? That is, can we think about h(100) as h(10^2) in the same manner?	d. Predict the value for h(25).
e. Make a generalization about the relationship between h(a^2) and h(a).	f. What is the relationship between h(a^3) and h(a)? Look at h(8) and h(2) for evidence.

g. What is the relationship between h(a^n) and h(a)? Look at h(16), h(32), and h(2) for evidence.

4. We've explored various products. What about quotients?

a. Note that $3 = 21 \div 7$. Can you find a relationship between h(3), h(21), and h(7)?

b. Test your theory on the number 6. Write 6 as a quotient in at least 2 different ways that will allow you to test your theory by using values in the table. Show your results. Do they agree?

c. Write a rule for quotients using good mathematical notation.

5. Thus far, we've only explored this function using positive, integral values of x. We'll now try to expand the domain of this function.

a. Find h(5.5) = h(11/2) using the rules you have already stated. How does it compare with h(5) and h(6)?

b. Find h(2/3). Will all numbers between 0 and 1 have similar function values? Why or why not?

c. Can you think of a way to find h(-1) using the rules we have come up with? What about h(0)?

6. a. Using your calculator, find $10^{h(9)}$. What do you notice?

b. Predict the value of $10^{h(42)}$. Use your calculator to test your prediction.

c. Why does h(100) = 2?

d. Try to explain the relationship between 10 and h(x) in words.

e. Can you think of a reason why negative numbers and zero are excluded from the domain of this function?

f. What is the common name of our mystery function? Use your calculator to evaluate some function values and compare the values to the values listed in the table.